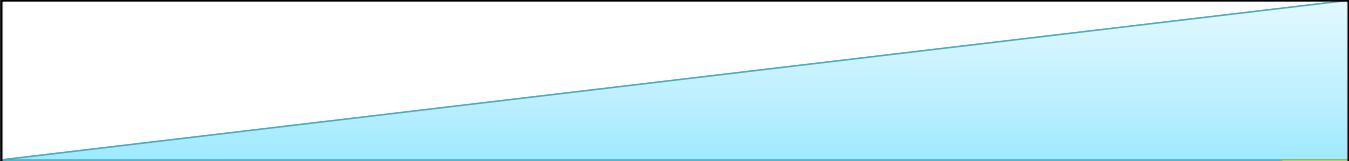
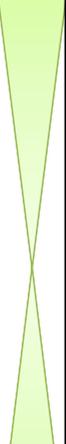


Congruent Triangles

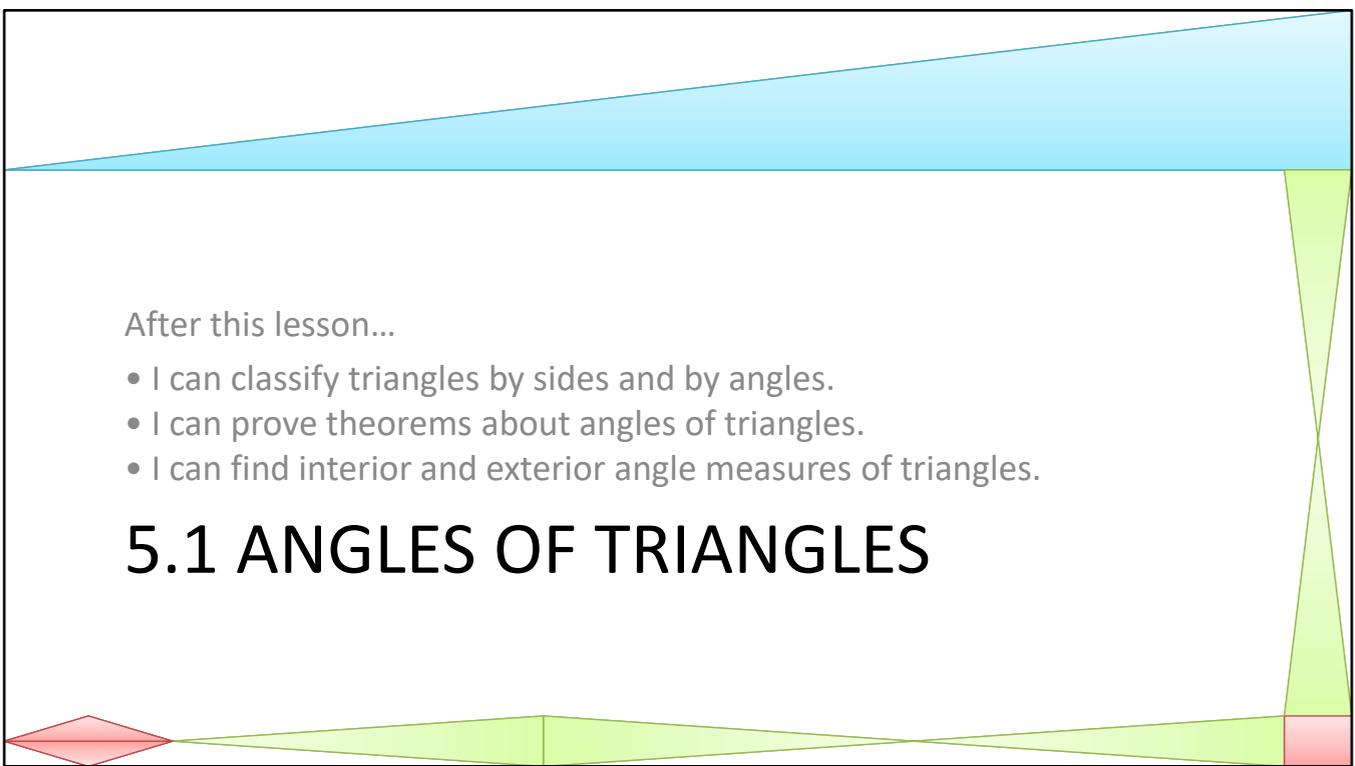
Geometry
Chapter 5

Geometry 5

- 
- This Slideshow was developed to accompany the textbook
 - *Big Ideas Geometry*
 - *By Larson and Boswell*
 - *2022 K12 (National Geographic/Cengage)*
 - Some examples and diagrams are taken from the textbook.



Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu



After this lesson...

- I can classify triangles by sides and by angles.
- I can prove theorems about angles of triangles.
- I can find interior and exterior angle measures of triangles.

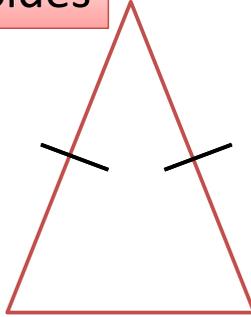
5.1 ANGLES OF TRIANGLES

5.1 Angles of Triangles

Classify Triangles by Sides



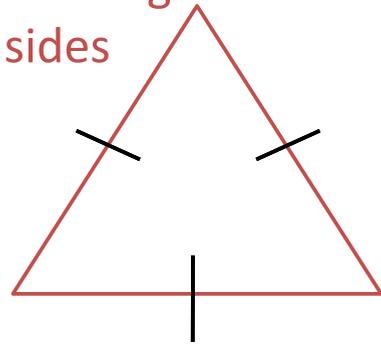
Scalene Triangle
No congruent sides



Isosceles Triangle
Two congruent sides

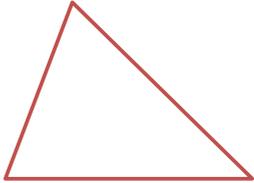
Equilateral Triangle

All congruent sides



5.1 Angles of Triangles

Classify Triangles by Angles



Acute Triangle
3 acute angles

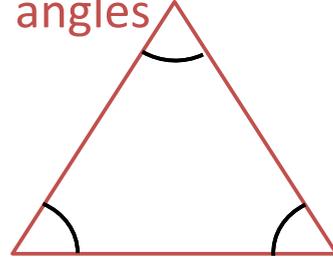


Right Triangle
1 right angle



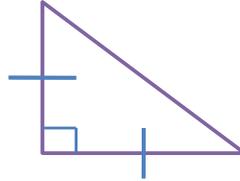
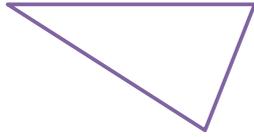
Obtuse Triangle
1 obtuse angle

Equiangular
Triangle
All congruent
angles



5.1 Angles of Triangles

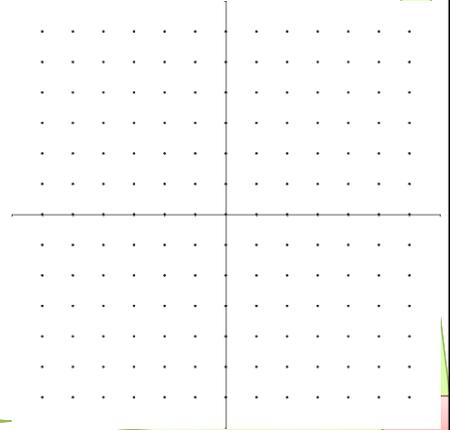
- Classify the following triangle by sides and angles



Scalene, Acute
Isosceles, Right

5.1 Angles of Triangles

- $\triangle ABC$ has vertices $A(0, 0)$, $B(3, 3)$, and $C(-3, 3)$. Classify it by its sides. Then determine if it is a right triangle.



Find length of sides using distance formula

$$AB = \sqrt{(3 - 0)^2 + (3 - 0)^2} = \sqrt{9 + 9} = \sqrt{18} \approx 4.24$$

$$BC = \sqrt{(-3 - 3)^2 + (3 - 3)^2} = \sqrt{(-6)^2 + 0} = \sqrt{36} = 6$$

$$AC = \sqrt{(-3 - 0)^2 + (3 - 0)^2} = \sqrt{9 + 9} = \sqrt{18} \approx 4.24$$

Isosceles

Check slopes to find right angles (perpendicular)

$$m_{AB} = (3 - 0)/(3 - 0) = 1$$

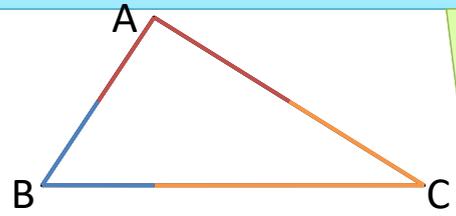
$$m_{BC} = (3 - 3)/(-3 - 3) = 0$$

$$m_{AC} = (3 - 0)/(-3 - 0) = -1$$

$AB \perp AC$ so it is a right triangle

5.1 Angles of Triangles

- Take a triangle and tear off two of the angles.
- Move the angles to the 3rd angle.
- What shape do all three angles form?



Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180° .

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

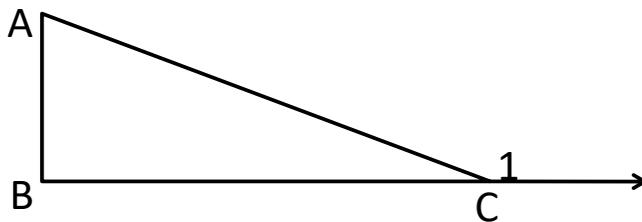
Straight line

5.1 Angles of Triangles

Exterior Angle Theorem

The measure of an exterior angle of a triangle = the sum of the 2 nonadjacent interior angles.

$$m\angle 1 = m\angle A + m\angle B$$



Proof:

$$m\angle A + m\angle B + m\angle ACB = 180^\circ \text{ (triangle sum theorem)}$$

$$m\angle 1 + m\angle ACB = 180^\circ \text{ (linear pair theorem)}$$

$$m\angle 1 + m\angle ACB = m\angle A + m\angle B + m\angle ACB \text{ (substitution)}$$

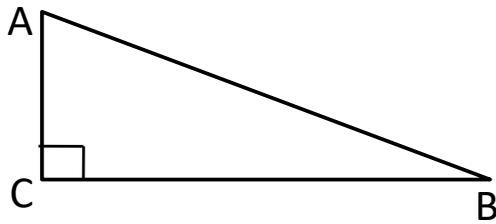
$$m\angle 1 = m\angle A + m\angle B \text{ (subtraction)}$$

5.1 Angles of Triangles

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

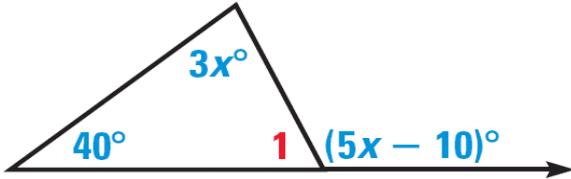
$$m\angle A + m\angle B = 90^\circ$$



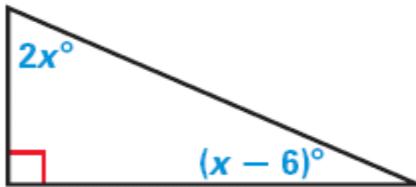
The proof involves saying that all three angles = 180. Since $m\angle C$ is 90, $m\angle A + m\angle B = 90$.

5.1 Angles of Triangles

- Find the measure of $\angle 1$ in the diagram.



- Find the measures of the acute angles in the diagram.



$$40 + 3x = 5x - 10 \rightarrow 50 = 2x \rightarrow x = 25$$

$$m\angle 1 + 40 + 3x = 180 \rightarrow m\angle 1 + 40 + 3(25) = 180 \rightarrow m\angle 1 + 40 + 75 = 180 \rightarrow m\angle 1 = 65$$

$$2x + x - 6 = 90 \rightarrow 3x = 96 \rightarrow x = 32$$

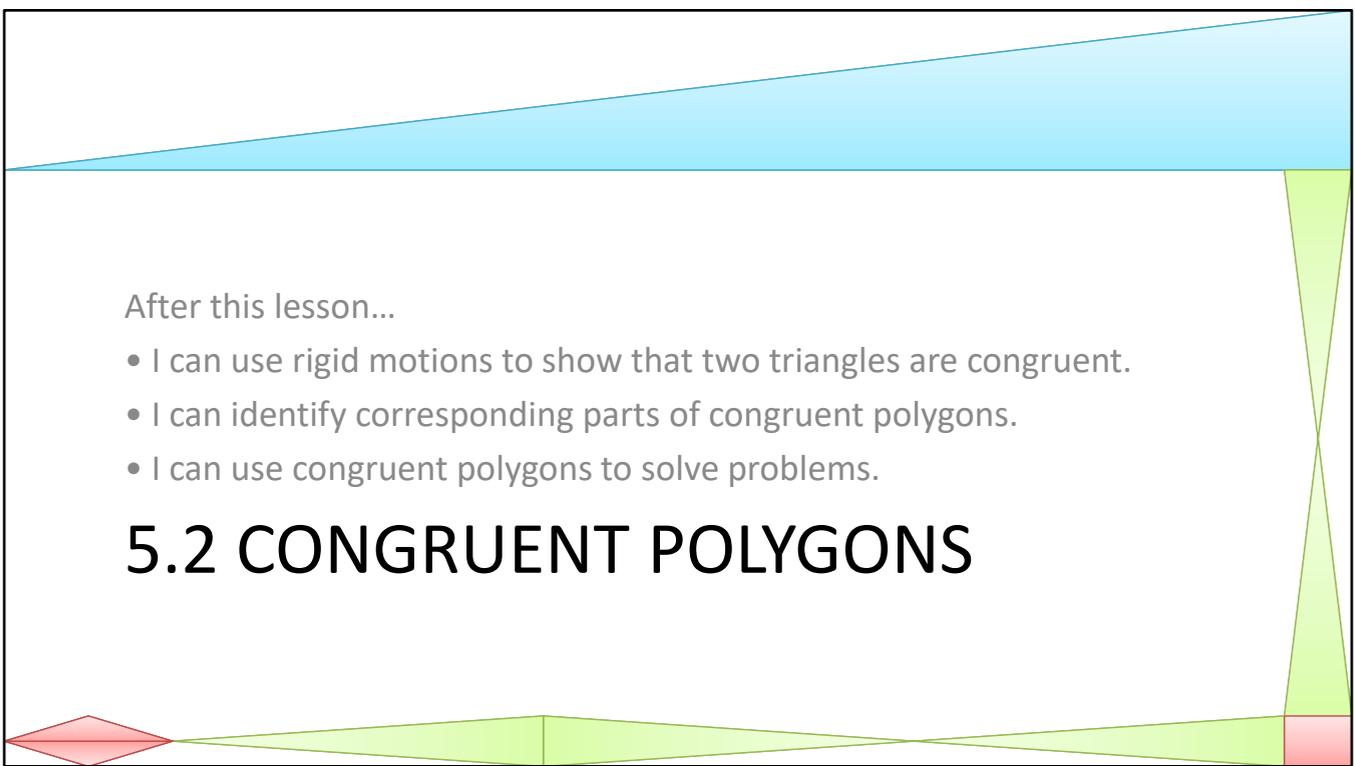
$$\text{Top angle: } 2x \rightarrow 2(32) = 64$$

$$\text{Angle at right: } x - 6 \rightarrow 32 - 6 = 26$$

5.1 Angles of Triangles

- 228 #2, 4, 6, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32, 42, 44, 48, 55, 58, 59 = 20 total





After this lesson...

- I can use rigid motions to show that two triangles are congruent.
- I can identify corresponding parts of congruent polygons.
- I can use congruent polygons to solve problems.

5.2 CONGRUENT POLYGONS

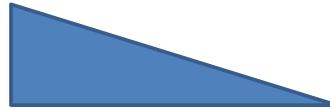
5.2 Congruent Polygons

Congruent \cong

Exactly the same shape and size.

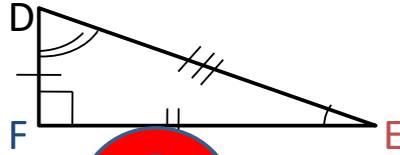
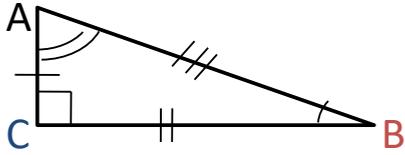


Congruent



Not Congruent

5.2 Congruent Polygons



- $\triangle ABC \cong \triangle DEF$

~~$\triangle ABC \cong \triangle DEF$~~

- $\angle A \cong \angle D$

$\angle B \cong \angle E$

$\angle C \cong \angle F$

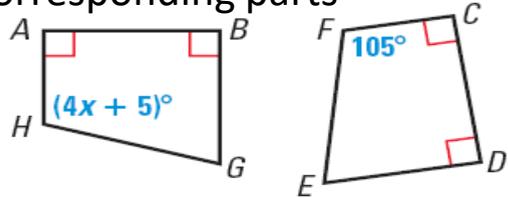
- $\overline{AB} \cong \overline{DE}$

$\overline{BC} \cong \overline{EF}$

$\overline{AC} \cong \overline{DF}$

5.2 Congruent Polygons

- In the diagram, $ABGH \cong CDEF$
 - Identify all the pairs of congruent corresponding parts
 - Find the value of x and find $m\angle H$.



$AB \cong CD, BG \cong DE, GH \cong EF, AH \cong CF$
 $\angle A \cong \angle C, \angle B \cong \angle D, \angle G \cong \angle E, \angle H \cong \angle F$

$$4x + 5 = 105$$

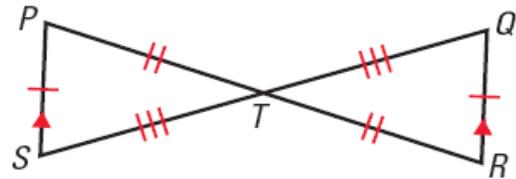
$$4x = 100$$

$$x = 25$$

$$m\angle H = 105^\circ$$

5.2 Congruent Polygons

- Show that $\triangle PTS \cong \triangle RTQ$

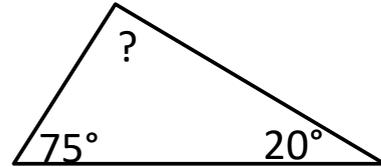
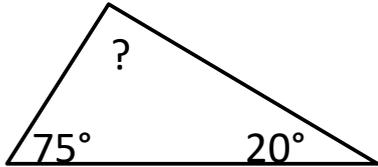


All of the corresponding parts of $\triangle PTS$ are congruent to those of $\triangle RTQ$ by the indicated markings, the Vertical Angle Theorem and the Alternate Interior Angle theorem.

5.2 Congruent Polygons

Third Angle Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.



Properties of Congruence of Triangles

Congruence of triangles is Reflexive, Symmetric, and Transitive

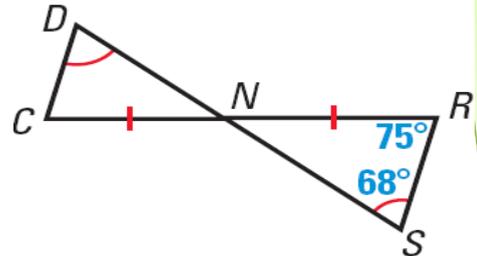
$$75 + 20 + ? = 180$$

$$95 + ? = 180$$

$$? = 85$$

5.2 Congruent Polygons

- In the diagram, what is $m\angle DCN$?



- By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$?

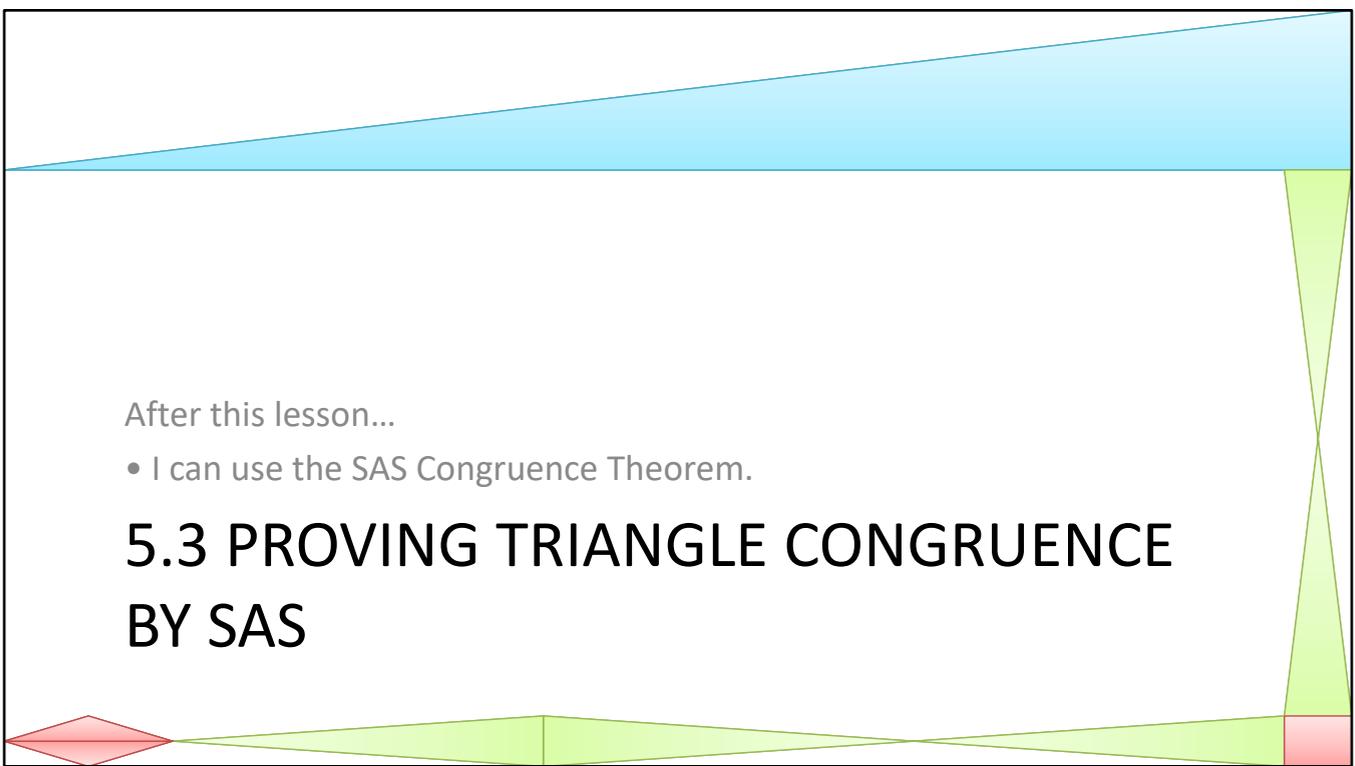
$m\angle DCN = 75^\circ$; alt int angle theorem (or 3rd angle theorem)

$DN \cong SN$, $DC \cong SR$

5.2 Congruent Polygons

- 235 #2, 3, 4, 6, 8, 10, 12, 13, 14, 15, 17, 18, 20, 21, 24, 26, 28, 30, 31, 32 = 20 total





After this lesson...

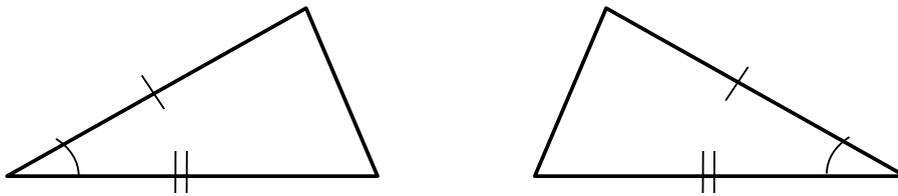
- I can use the SAS Congruence Theorem.

5.3 PROVING TRIANGLE CONGRUENCE BY SAS

5.3 Proving Triangle Congruence by SAS

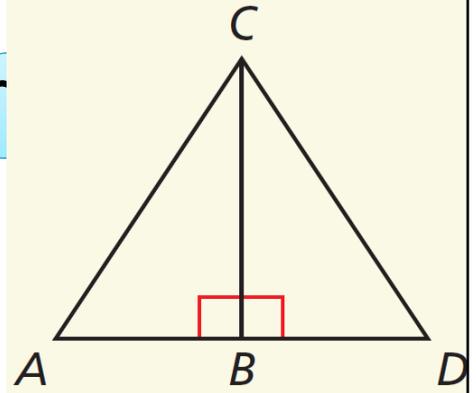
SAS (Side-Angle-Side Congruence Postulate)

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent



The angle must be between the sides!!!

5.3 Proving Triangle Congruence



- **Given** B is the midpoint of \overline{AD} .
 $\angle ABC$ and $\angle DBC$ are right angles.
- **Prove** $\triangle ABC \cong \triangle DBC$

Statements

Reasons

B is the midpoint of \overline{AD} . $\angle ABC$ and $\angle DBC$ are right angles (given)

$\overline{AB} \cong \overline{BD}$ (definition of midpoint)

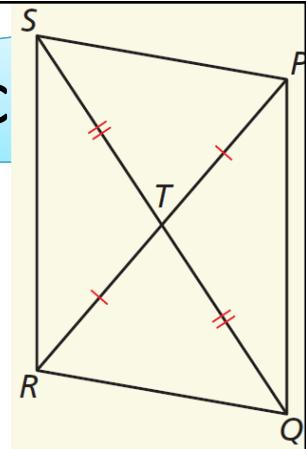
$\angle ABC \cong \angle DBC$ (rt. Angles are \cong)

$\overline{BC} \cong \overline{BC}$ (reflexive)

$\triangle ABC \cong \triangle DBC$ (SAS)

5.3 Proving Triangle Congruence

- What can you conclude about $\triangle PTS$ and $\triangle RTQ$? Explain.

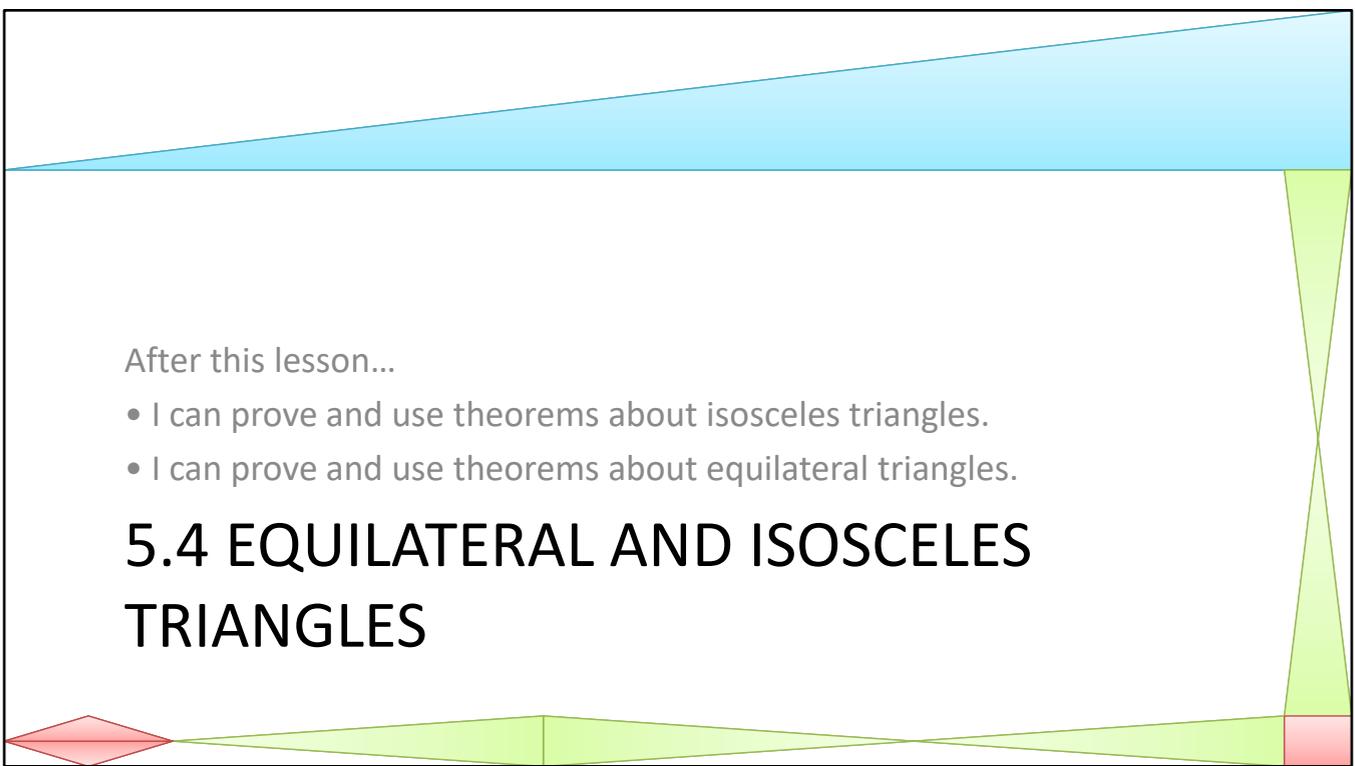


- 241 #2, 4, 6, 7, 8, 10, 12, 17, 18, 19, 23, 24, 27, 29, 31 = 15 total

$PT \cong TR$ and $ST \cong TQ$ (given)

$\angle PTS \cong \angle RTQ$ (vertical angles are \cong)

$\triangle PTS \cong \triangle RTQ$ (SAS)



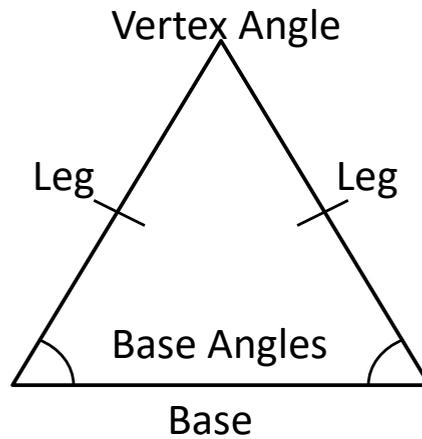
After this lesson...

- I can prove and use theorems about isosceles triangles.
- I can prove and use theorems about equilateral triangles.

5.4 EQUILATERAL AND ISOSCELES TRIANGLES

5.4 Equilateral and Isosceles Triangles

- Parts of an Isosceles Triangle



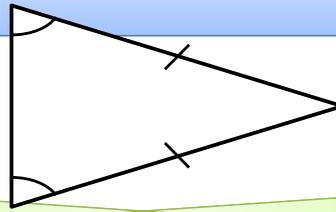
5.4 Equilateral and Isosceles Triangles

Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

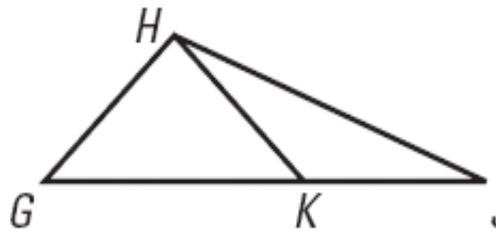
Converse of Base Angles Theorem

If two angles of a triangle are congruent, then the two sides opposite them are congruent.



5.4 Equilateral and Isosceles Triangles

- Complete the statement
 - If $\overline{HG} \cong \overline{HK}$, then $\angle \underline{\quad ? \quad} \cong \angle \underline{\quad ? \quad}$.
 - If $\angle KHJ \cong \angle KJH$, then $\underline{\quad ? \quad} \cong \underline{\quad ? \quad}$.



$$\angle HKG \cong \angle HGK$$

$$KJ \cong KH$$

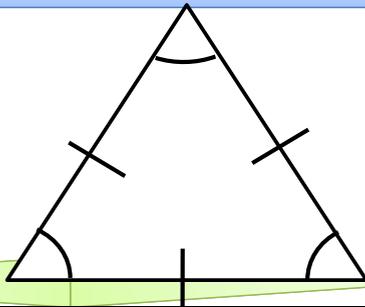
5.4 Equilateral and Isosceles Triangles

Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

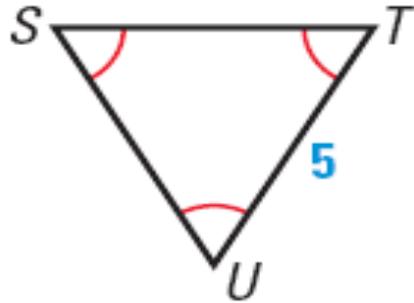
Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



5.4 Equilateral and Isosceles Triangles

- Find ST
- Find $m\angle T$

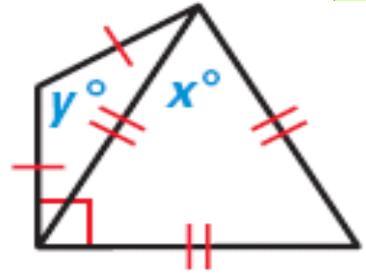


$$ST = 5$$

$$m\angle T = 60^\circ \text{ (all angles in equilateral/equiangular triangles are } 60^\circ\text{)}$$

5.4 Equilateral and Isosceles Triangles

- Find the values of x and y

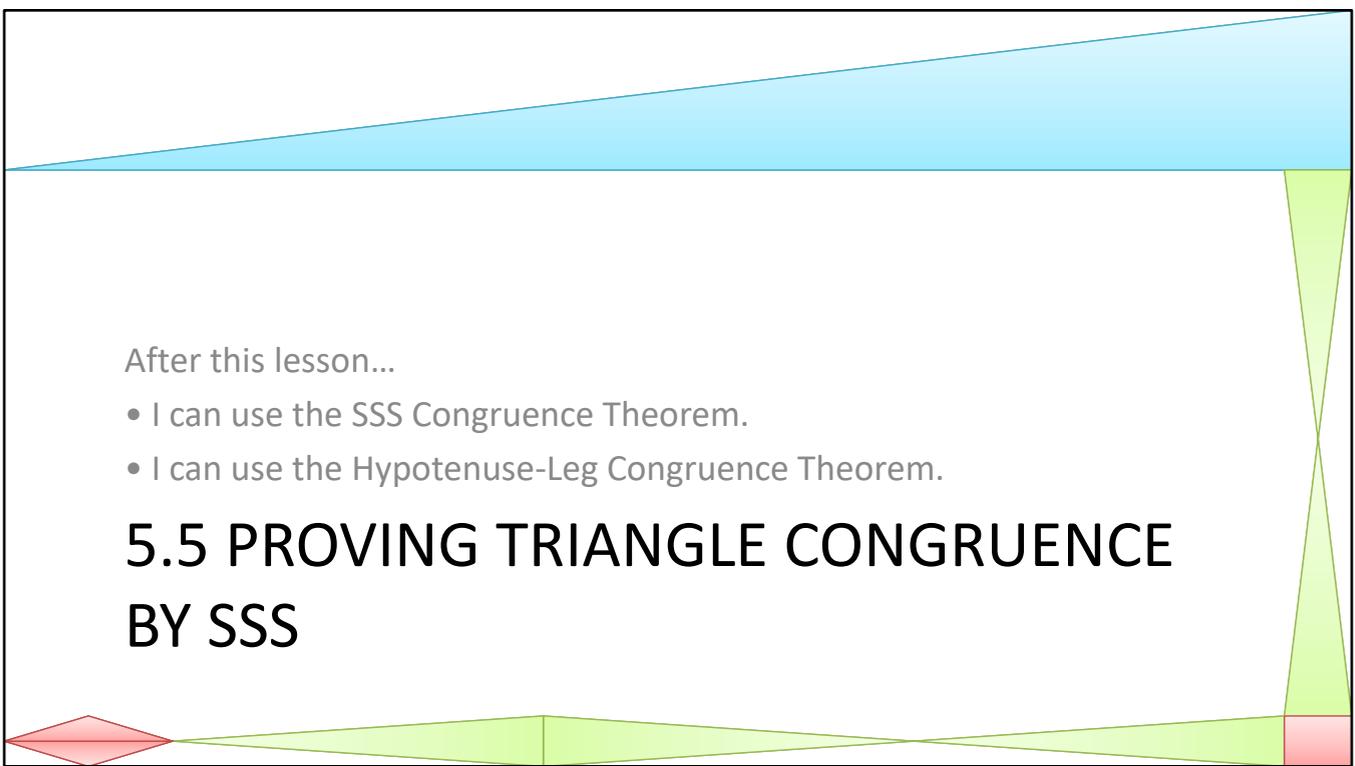


- 248 #2, 4, 6, 8, 12, 14, 16, 18, 20, 21, 22, 24, 27, 28, 30, 36, 38, 39, 40, 43 = 20 total

$x = 60$; equilateral triangle

Each base angle by y ; $60 + ? = 90 \rightarrow ? = 30$

Angle sum theorem: $30 + 30 + y = 180 \rightarrow y = 120$



After this lesson...

- I can use the SSS Congruence Theorem.
- I can use the Hypotenuse-Leg Congruence Theorem.

5.5 PROVING TRIANGLE CONGRUENCE BY SSS

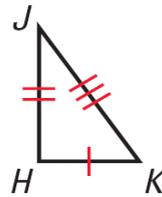
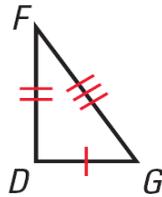
5.5 Proving Triangle Congruence by SSS

SSS (Side-Side-Side Congruence Postulate)

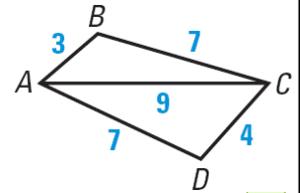
If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent

- True or False

- $\triangle DFG \cong \triangle HJK$



- $\triangle ACB \cong \triangle CAD$

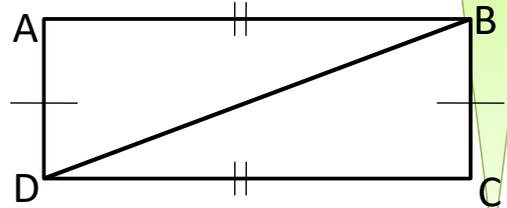


True

False

5.5 Proving Triangle Congruence by SSS

- Given: $\overline{AB} \cong \overline{DC}$; $\overline{AD} \cong \overline{BC}$
- Prove: $\triangle ABD \cong \triangle CDB$



Statements

Reasons

$AB \cong DC$; $AD \cong CB$

$BD \cong BD$

$\triangle ABD \cong \triangle CDB$

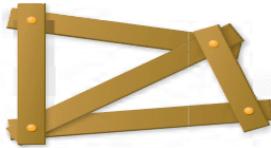
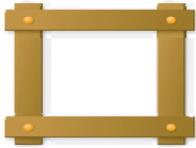
(given)

(reflexive)

(SSS)

5.5 Proving Triangle Congruence by SSS

- Stable structures are made out of triangles
- Determine whether the figure is stable.



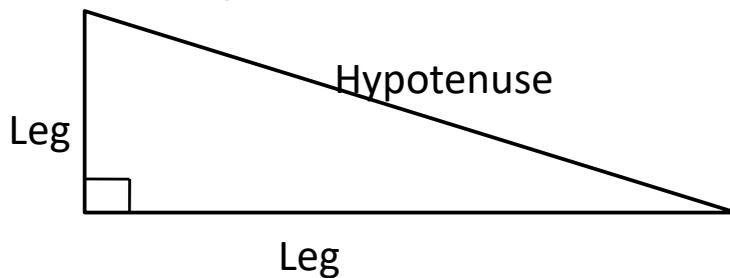
Not stable

Stable since has triangular construction

Not stable, lower section does not have triangular construction

5.5 Proving Triangle Congruence by SSS

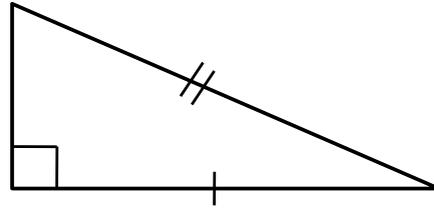
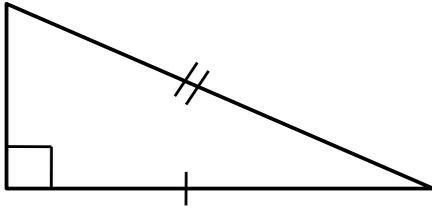
- Right triangles are special
 - If we know two sides are congruent we can use the Pythagorean Theorem (ch 7) to show that the third sides are congruent



5.5 Proving Triangle Congruence by SSS

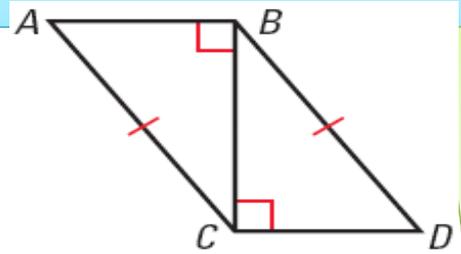
HL (Hypotenuse-Leg Congruence Theorem)

If the hypotenuse and a leg of a **right** triangle are congruent to the hypotenuse and a leg of another **right** triangle, then the two triangles are congruent



5.5 Proving Triangle Congruence by SSS

- Given: $\angle ABC$ and $\angle BCD$ are rt \angle s; $\overline{AC} \cong \overline{BD}$
- Prove: $\triangle ACB \cong \triangle DBC$



Statements	Reasons
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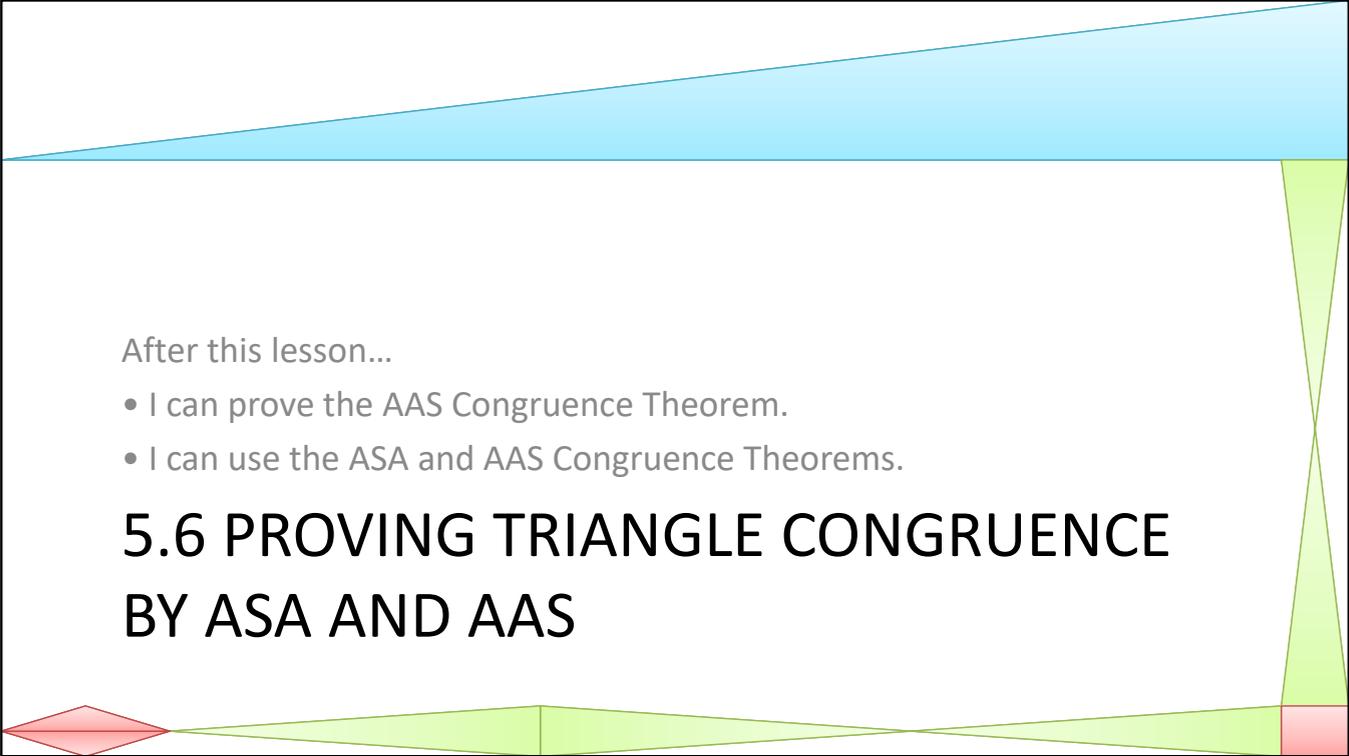
$\angle ABC$ and $\angle BCD$ are rt \angle s; $AC \cong BD$	
$\triangle ACB$ and $\triangle DBC$ are rt \triangle	
$BC \cong CB$	
$\triangle ACB \cong \triangle DBC$	(HL)

(given)
(def rt \triangle)
(reflexive)

5.5 Proving Triangle Congruence by SSS

- 256 #1, 2, 3, 4, 6, 7, 8, 10, 12, 14, 18, 20, 22, 26, 28, 31, 32, 34, 35, 36 = 20 total





After this lesson...

- I can prove the AAS Congruence Theorem.
- I can use the ASA and AAS Congruence Theorems.

5.6 PROVING TRIANGLE CONGRUENCE BY ASA AND AAS

5.6 Proving Triangle Congruence by ASA and AAS

- Use a ruler to draw a line of 5 cm.
- On one end of the line use a protractor to draw a 30° angle.
- On the other end of the line draw a 60° angle.
- Extend the other sides of the angles until they meet.
- Compare your triangle to your neighbor's.
- This illustrates ASA.

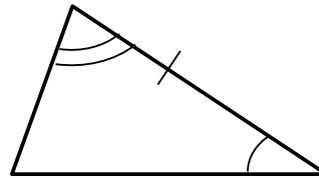
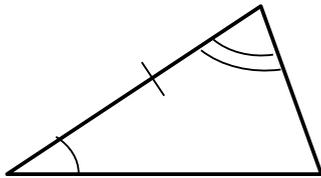


Everyone's triangle should be congruent

5.6 Proving Triangle Congruence by ASA and AAS

ASA (Angle-Side-Angle Congruence Postulate)

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent

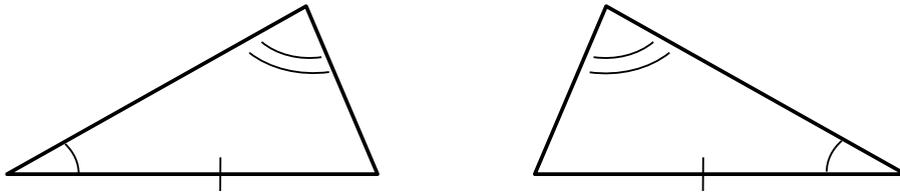


The side must be between the angles!

5.6 Proving Triangle Congruence by ASA and AAS

AAS (Angle-Angle-Side Congruence Theorem)

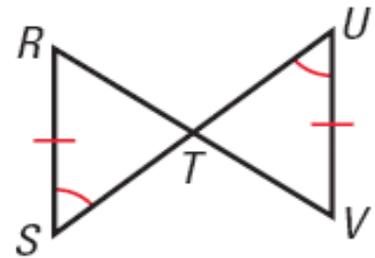
If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, then the two triangles are congruent



The side is
NOT
between
the angles!

5.6 Proving Triangle Congruence by ASA and AAS

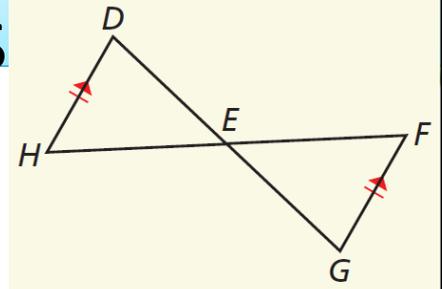
- In the diagram, what postulate or theorem can you use to prove that $\triangle RST \cong \triangle VUT$?



$\angle RTS \cong \angle UTV$ by Vert. Angles are Congruent
 $\triangle RST \cong \triangle VUT$ by AAS

5.6 Proving Triangle Congruence by ASA and AAS

- **Given:** $\overline{DH} \parallel \overline{GF}$, $\overline{DE} \cong \overline{GE}$
- **Prove:** $\triangle DEH \cong \triangle GEF$



Statements

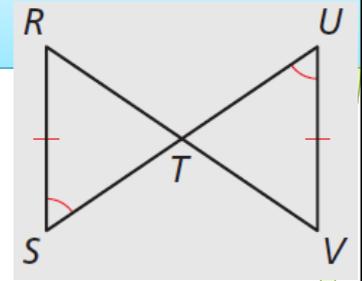
Reasons

$\overline{DH} \parallel \overline{GF}$, $\overline{DE} \cong \overline{GE}$
 $\angle D \cong \angle G$, $\angle H \cong \angle F$
 $\triangle DEH \cong \triangle GEF$

(given)
 (alt int angles thm)
 (ASA)

5.6 Proving Triangle Congruence by ASA and AAS

- **Given:** $\overline{RS} \cong \overline{VU}$, $\angle S \cong \angle U$
- **Prove:** $\triangle RST \cong \triangle VUT$



Statements

Reasons

$$\overline{RS} \cong \overline{VU}, \angle S \cong \angle U$$

$$\angle RTS \cong \angle VTU$$

$$\triangle RST \cong \triangle VUT$$

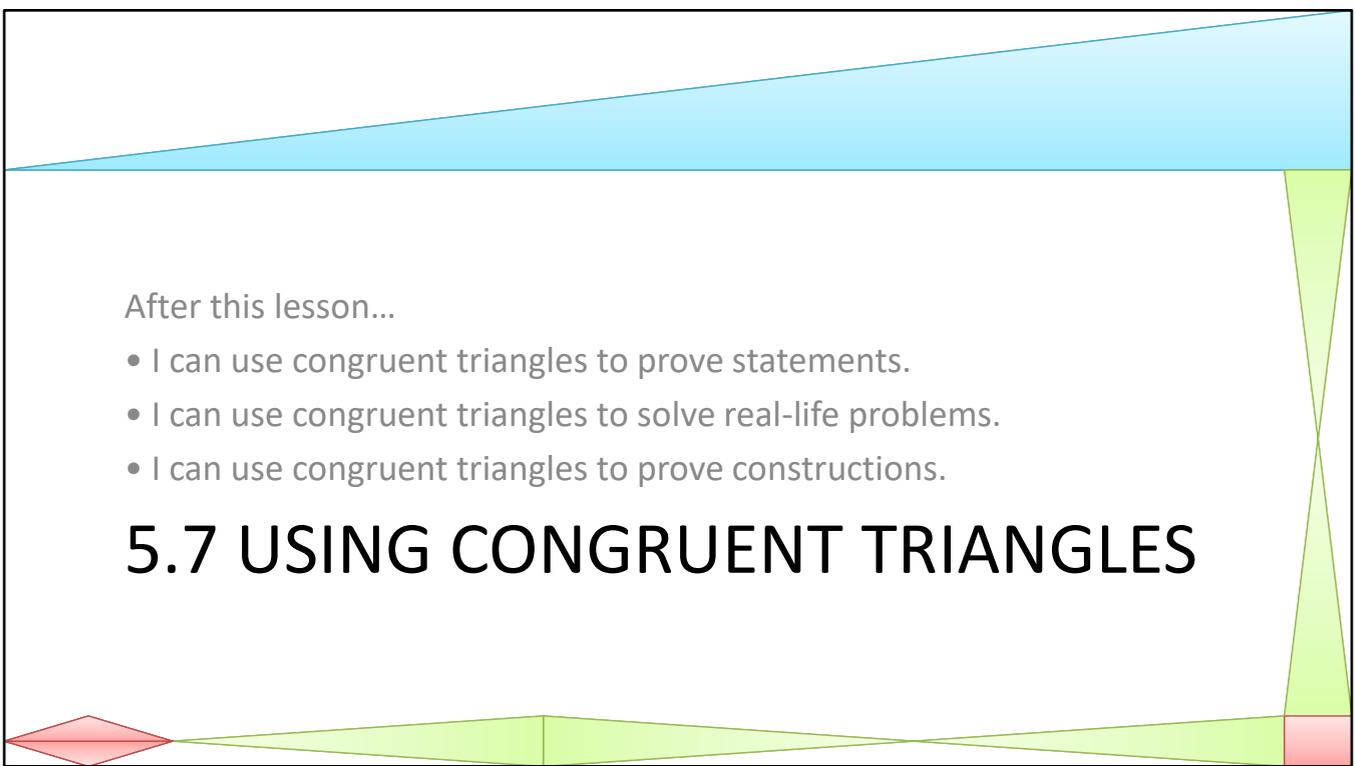
(given)

(vert angles \cong)

(AAS)

5.6 Proving Triangle Congruence by ASA and AAS

- 264 #2, 4, 6, 8, 12, 14, 16, 22, 24, 28, 35, 38, 39, 40, 41 = 15 total



After this lesson...

- I can use congruent triangles to prove statements.
- I can use congruent triangles to solve real-life problems.
- I can use congruent triangles to prove constructions.

5.7 USING CONGRUENT TRIANGLES

5.7 Using Congruent Triangles

- By the definition of congruent triangles, we know that the corresponding parts have to be congruent

CPCTC

Corresponding Parts of Congruent Triangles are Congruent

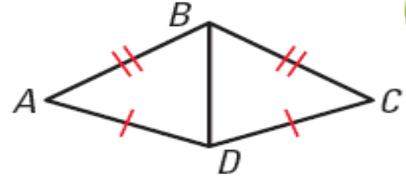
Your book just calls this “definition of congruent triangles”

5.7 Using Congruent Triangles

- To show that parts of triangles are congruent
 - First show that the triangles are congruent using
 - SSS, SAS, ASA, AAS, HL
 - Second say that the corresponding parts are congruent using
 - CPCTC or “def $\cong \Delta$ ”

5.7 Using Congruent Triangles

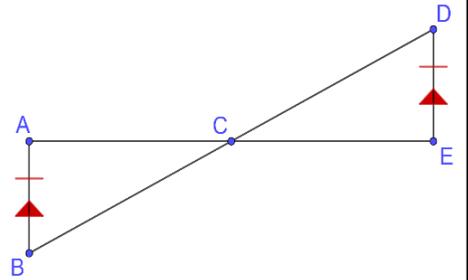
- Write a plan for a proof to show that $\angle A \cong \angle C$



- Show that $\overline{BD} \cong \overline{BD}$ by reflexive
- Show that triangles are \cong by SSS
- Say $\angle A \cong \angle C$ by def $\cong \Delta$ or CPCTC

5.7 Using Congruent Trian

- Given: $\overline{AB} \cong \overline{DE}$, $\overline{AB} \parallel \overline{DE}$
- Prove: C is the midpoint of \overline{AE}



$$\overline{AB} \cong \overline{DE}, \overline{AB} \parallel \overline{DE}$$

$$\angle B \cong \angle D, \angle A \cong \angle E$$

$$\triangle ABC \cong \triangle EDC$$

$$\overline{AC} \cong \overline{CE}$$

C is midpoint of \overline{AE}

(given)

(Alt. Int. \angle Thrm)

(ASA)

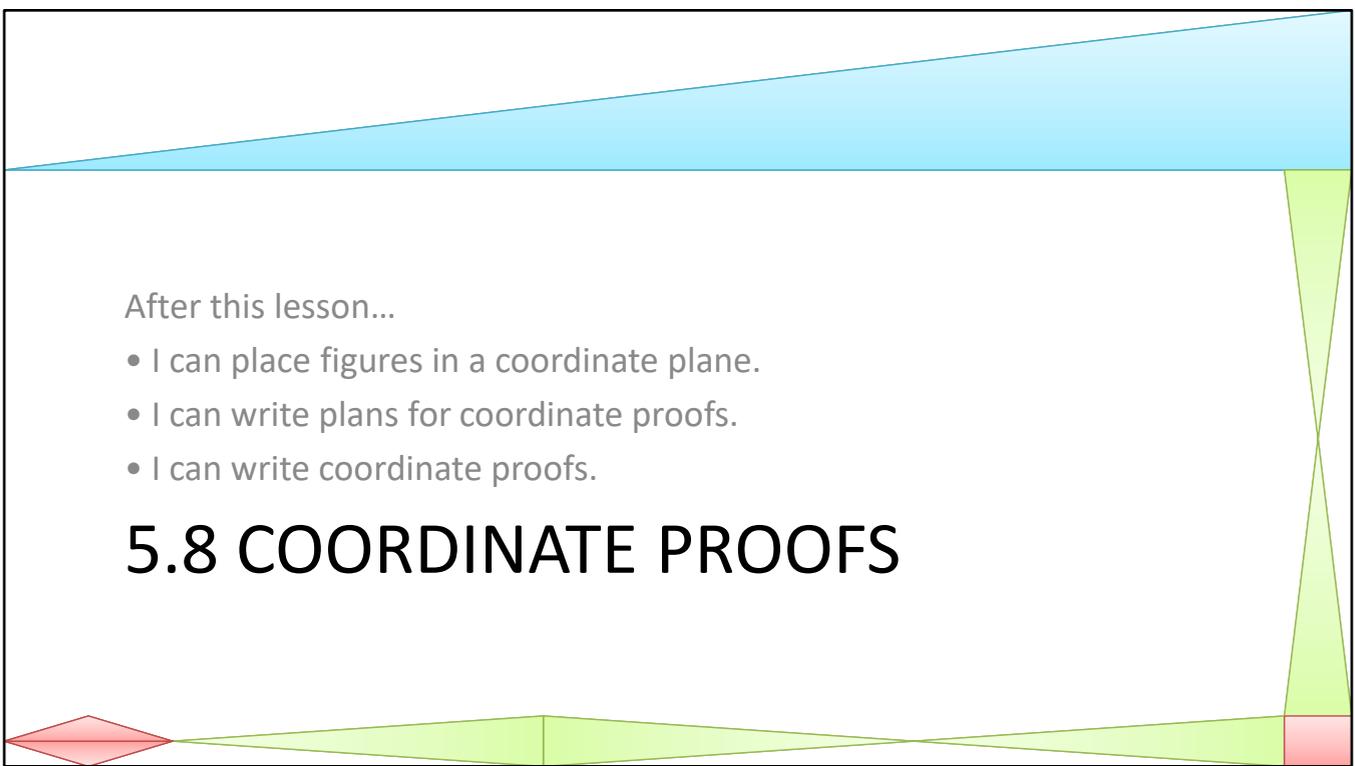
(CPCTC)

(Def midpoint)

5.7 Using Congruent Triangles

- 271 #2, 3, 4, 6, 8, 10, 13, 17, 19, 20, 23, 25, 26, 27, 28 = 15 total





After this lesson...

- I can place figures in a coordinate plane.
- I can write plans for coordinate proofs.
- I can write coordinate proofs.

5.8 COORDINATE PROOFS

5.8 Coordinate Proofs

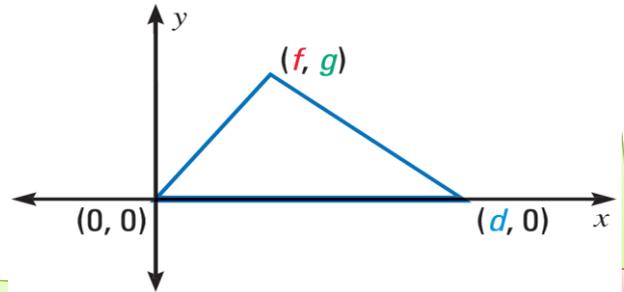
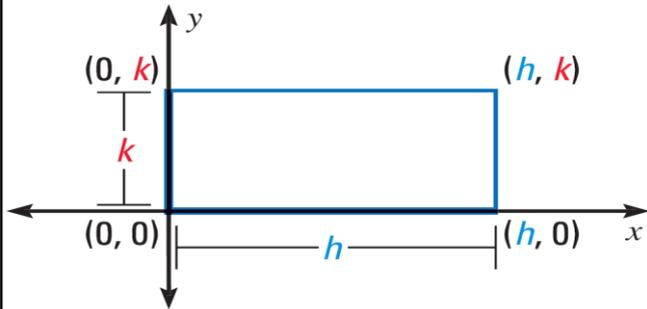
Coordinate Proof

- Place geometric figures in a coordinate plane (graph)
- When variables are used for the coordinates, the result is true for all figures of that type
- Use formulas to prove things
 - Midpoint formula
 - Distance formula
 - Slope formula

5.8 Coordinate Proofs

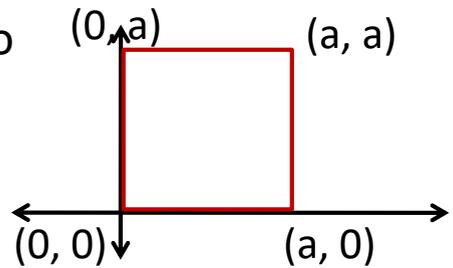
- Place figures for Coordinate Proof
 1. Use the origin as a vertex or center.
 2. Place at least one side of the polygon on an axis.
 3. Usually keep the figure within the first quadrant.
 4. Use coordinates that make computations as simple as possible.
- You will prove things by calculating things like slope, distance, and midpoints

5.8 Coordinate Proofs

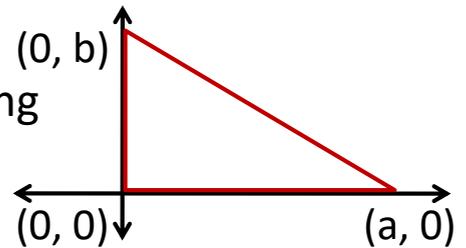


5.8 Coordinate Proofs

- Place a **square** in a coordinate plane so that it is convenient for finding side lengths. Assign coordinates.

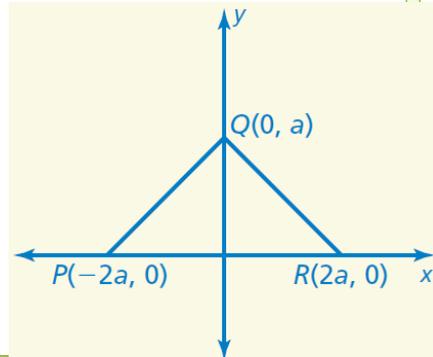


- Place a **right triangle** in a coordinate plane so that it is convenient for finding side lengths. Assign coordinates.



5.8 Coordinate Proofs

- Place an isosceles triangle in a coordinate plane with vertices $P(-2a, 0)$, $Q(0, a)$, and $R(2a, 0)$. Then find the side lengths and the coordinates of the midpoint of each side.



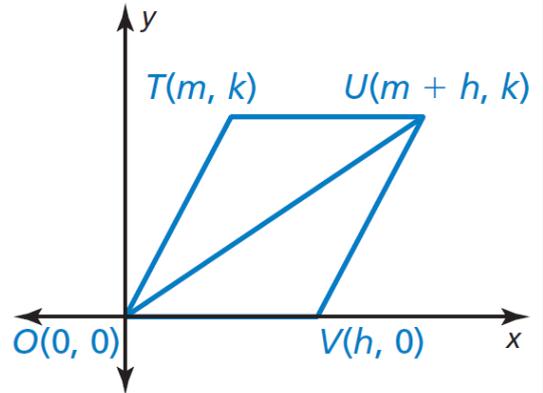
$$PQ = \sqrt{(0 - (-2a))^2 + (a - 0)^2} = \sqrt{4a^2 + a^2} = \sqrt{5a^2} = a\sqrt{5}; M_{\overline{PQ}} \left(\frac{-2a + 0}{2}, \frac{0 + a}{2} \right) \\ = \left(-a, \frac{a}{2} \right)$$

$$QR = \sqrt{(0 - 2a)^2 + (a - 0)^2} = \sqrt{4a^2 + a^2} = \sqrt{5a^2} = a\sqrt{5}; M_{\overline{QR}} \left(\frac{2a + 0}{2}, \frac{0 + a}{2} \right) \\ = \left(a, \frac{a}{2} \right)$$

$$PR = \sqrt{(2a - (-2a))^2 + (0 - 0)^2} = \sqrt{16a^2} = 4a; M_{\overline{PR}} \left(\frac{2a + (-2a)}{2}, \frac{0 + 0}{2} \right) = (0, 0)$$

5.8 Coordinate Proofs

- **Given** Coordinates of vertices of quadrilateral $OTUV$
- **Prove** $\angle TOU \cong \angle VUO$



- 277 #2, 4, 6, 8, 11, 12, 15, 16, 22, 23, 25, 26, 29, 32, 33 = 15 total

The slope of \overline{TO} is $\frac{k-0}{m-0} = \frac{k}{m}$.

The slope of \overline{UV} is $\frac{k-0}{(m+h)-h} = \frac{k}{m}$.

Because they have the same slope, $\overline{TO} \parallel \overline{UV}$.

By the Alternate Interior Angles Theorem, $\angle TOU \cong \angle VUO$.